# ANSWERS TO SELECTED EXERCISES

## Chapter 1

SECTION 1.1

1. (a) $(1, -7)$
   (b) $(4, 3/4)$
   (c) $(8, 6, 1)$
   (d) $(0, 1, 2, 1)$
   (e) $(11, 13, -2)$
   (f) $(4 - 2y - 3z, y, z)$
   (g) $(4, 3)$
   (h) $(5, 2)$
   (i) $(5 - y, y, -1 - w, w)$
   (j) no solutions

2. (a) $(-z, 0, z)$
   (b) $(0, -z, z)$
   (c) $(-z, -w, z, w)$

5. (a) $\begin{pmatrix} 1 & -6 & -4 \\ 0 & 1 & -11 \\ 0 & 0 & 1 \end{pmatrix}$ (c) $\begin{pmatrix} 1 & 2 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

   (b) $\begin{pmatrix} 1 & 0 & 0 & 6 & 5 \\ 0 & 1 & 0 & -4 & -10/3 \\ 0 & 0 & 1 & 7 & 4 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$

6. (a) (32/78, −5/78, 35/78)
   (b) $(-3-5x^5, 2+(10/3)x^5, -(7/2)-4x^5, 1/2, x^5)$
   (c) $(-3-5x^5, 8/3+(10/3)x^5, -4-4x^5, 1, x^5)$
   (d) no solutions
   (e) no solutions

SECTION 1.2

11. (a) (120/52, 4/52)
    (b) (4/5, −8/5)
    (c) (−51/22, 29/22)
    (d) (−39/5, −43/5)
12. (a) $3x+7y=-1$
    (b) $x-y=8$
    (c) $y-2x=14$

SECTION 1.3

13. (a) $\begin{pmatrix} 1 & 0 & 0 \\ 1 & -2 & 1 \\ 1 & 0 & 1 \end{pmatrix}$ (c) (24)

    (b) $\begin{pmatrix} 0 & 0 & 0 & 0 \\ 12 & 15 & 14 & 7 \\ -4 & -5 & 2 & 5 \\ 3 & 19 & -1 & 9 \end{pmatrix}$ (d) $\begin{pmatrix} 8 & 10 \\ 2 & 1 \\ 1 & 0 \\ -1 & -2 \end{pmatrix}$

14. (a) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & -2 & -2 \\ 1 & 0 & 1 \end{pmatrix}$ (c) $\begin{pmatrix} 24 & 24 & 12 & 8 & 4 \\ 12 & 12 & 6 & 4 & 2 \\ -6 & -6 & -3 & -2 & -1 \\ 48 & 48 & 24 & 16 & 8 \\ 42 & 42 & 21 & 14 & 7 \end{pmatrix}$

    (b) $\begin{pmatrix} 23 & -7 & 10 & -5 \\ -3 & -4 & -16 & -2 \\ 3 & 9 & 12 & 5 \\ 23 & -7 & 10 & -5 \end{pmatrix}$

    (d) doesn't exist

15. (a) $\begin{pmatrix} 0 & 0 & -1 \\ 3 & -1 & -3 \\ -20/78 & 7/78 & 21/78 \end{pmatrix}$ (c) $\begin{pmatrix} 0 & -1/2 & 0 & 0 \\ 1/6 & 1/12 & 0 & 0 \\ -1/6 & 5/12 & 1/8 & 0 \\ -1/2 & 1/4 & 0 & 1/3 \end{pmatrix}$

(b) $$\begin{pmatrix} 1 & 0 & 0 & 0 \\ -1 & 1/3 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1/2 \end{pmatrix}$$

16. (a) no conditions
(b) $b^1 = 0$
(c) $2b^2 + b^3 + b^4 = 0$
$b^1 + 3b^2 - 2b^4 = 0$
$4b^2 - b^4 + b^5 = 0$
(d) $-4b^1 - 25b^2 + 14b^3 + 10b^4 = 0$

17. Since the index $d$ of $A$ is at most $n$, if $P$ row reduces $A$ we obtain $PAx = Pb$. Since (at least) the last $m - d$ rows of $PA$ are zero, $b$ must satisfy the (non-vacuous) conditions that the last $m - d$ entries of $Pb$ are zero.

19. If $x = \sum x^i E_i$, $T(x) = \sum x^i T(E_i) = 0$.

20. If $x = \sum x^i E_i$, $T(x) = \sum_{i=1}^{n-1} x^i E_{i+1} + x^n E_1$, so $T$ is uniquely determined by the conditions.

SECTION 1.4

21. (a) 4 (b) 3 (c) 3 (d) 3
22. (a) 3 (b) 3 (c) 3
23. (a) $\{x \in R^4;\ x^1 + x^2 = x^3 + x^4\}$
(b) $\{x \in R^4;\ -3x^1 + x^3 = 0,\ 2x^1 - x^2 - x^4 = 0\}$
(c) $\{x \in R^3;\ x^1 + 2x^2 - x^3 = 0\}$
(d) $\{x \in R^5;\ x^3 = x^5,\ x^2 = 0,\ x^4 = 0\}$

24. (a) No (b) Yes (c) No

25. (a) $c(-4v_1 - v_2 - 6v_3 + 5v_4) = 0$
(b) $a(v_2 - v_4) + b(2v_1 + v_3 + v_4) = 0$
(c) $av_1 + b(2v_2 - 2v_3 + v_4) = 0$

26. The given vectors form a basis for $R^5$.
27. (a) $(0, -1/2, 1, 0, 0)$ $(-1, 1/2, 0, 0, 1)$
(b) $(1, 2, 1, 0)$ $(0, -1, 0, 1)$

SECTION 1.5

29. (a) $K = \{6x^1 = 17x^4,\ x^2 = -2x^4,\ x^3 = x^4/3\}$
$R = R^3$
(b) $K = \{x^1 = 0,\ x^2 = 0\}$
$R = \{6x^2 = 12x^4 - 11x^1,\ 2x^3 = 4x^4 - 39x^1\}$

(c) $K = \{x^1 = 0,\ 3x^2 + 2x^4 = 0,\ 3x^3 + 4x^4 = 0\}$
$R = \{x^1 - x^2 + x^3 - x^4 = 0\}$
(d) $K = \{8x^1 + x^4 + 6x^5 = 0,\ 8x^2 + 5x^4 + 7x^5 = 0,\ x^3 + 2x^4 + 2x^5 = 0\}$
$R = R^3$

30. (a) $K$: $(17/6, -2, 1/3, 1)$, $R$: $E_1, E_2, E_3$
(b) $K$: $(0, 0, 1, 0), (0, 0, 0, 1)$, $R$: $(1, -11/6, -39/2, 0), (0, 2, 4, 1)$
(c) $K$: $(0, -2/3, -4/3, 1)$, $R$: $(1, 1, 0, 0), (-1, 0, 1, 0), (1, 0, 0, 1)$
(d) $K$: $(-1, -5, -16, 8, 0), (-3, -7, -8, 0, 4)$, $R$: $E_1, E_2, E_3$

31. If $f$ is nonzero, its range is all of $R$, so its rank is 1. Thus its nullity is $n - 1$.

32. $\{E_1 - E_i\colon\ i = 2, \ldots, n\}$

SECTION 1.6

33. (a) $\frac{1}{2}\begin{pmatrix} 0 & -2 & 1 \\ 2 & 4 & -2 \\ 0 & 0 & 1 \end{pmatrix}$

(b) $\frac{1}{12}\begin{pmatrix} -1 & -1 & -7 \\ 11 & -1 & -8 \\ -7 & 5 & 4 \end{pmatrix}$

(c) $\begin{pmatrix} 0 & 1 & 1 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$

(d) $\begin{pmatrix} 1/9 & 1/9 & -1/3 & 1/9 \\ 1/6 & 0 & 1/6 & -1/12 \\ -1/9 & 2/9 & 0 & -1/9 \\ -2/9 & -2/9 & 6/9 & 5/18 \end{pmatrix}$

34. (a) $(0, 2, 1)$ (c) $(9/8, 1/2, -7/8)$
(b) $(-17, 5, 1)$ (d) $(-1/2, 1/2, 1)$

35. By induction we can show that $A^k$ has the property that its $(i, j)$ entries are zero for all $i \leq j + k - 1$. Once $k \geq n$, these are all the entries.

36. $(I + A)^{-1} = \sum_{i=0}^{n=1} (-1)^i A^i$

SECTION 1.7

37.

| Eigenvalue | | Eigenvectors |
|---|---|---|
| (a) | 2 | $(0, 1, -2)$ |
| | 3 | $(1, -2, -4)$ |
| | $-1$ | $(1, 1, -4)$ |

| | | |
|---|---|---|
| (b) | 1 | (1, 0, 1, −1) |
| | −1 | (2, 0, 2, −3) |
| | 0 | (0, 0, 1, −1) |
| | 2 | (0, 2, 0, 0) |
| (c) | 1 | (1, 0, 0, 0), (0, 0, 1, −1) |
| | 4 | (0, 1, 0, 0) |
| | no basis of eigenvectors | |
| (d) | 2 | (1, 0, 1), (0, 1, 1) |
| | −2 | (1, 1, −2) |

39. If $G$ represents the standard basis, we use Exercise 38 to find $A_G{}^F$, $A_G{}^E$ and use the fact that

$$A_E{}^F = A_G{}^F(A_G{}^E)^{-1}$$

(a) $\frac{1}{9}\begin{pmatrix} 2 & 5 & -1 \\ 5 & 1 & 2 \\ -2 & 4 & 1 \end{pmatrix}$ (c) $\begin{pmatrix} 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1/2 \\ 0 & -1 & 0 & 1 \\ 1/2 & -1/2 & 1/2 & -1/2 \end{pmatrix}$

(b) $\begin{pmatrix} 1/2 & -1/2 & -1/2 \\ 3/16 & -5/16 & -7/16 \\ -1/2 & 1/2 & -1/2 \end{pmatrix}$

40. (a) $\begin{pmatrix} 0 & -2 & 2 \\ 2 & 3 & -1 \\ 2 & 2 & 0 \end{pmatrix}$ (b) $\begin{pmatrix} -3 & -5 & 0 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{pmatrix}$

41. If $T_E = cI$, when $E$ is a basis of eigenvectors, then for any basis $F$,

$$T_F = (A_E{}^F)^{-1} T_E A_E{}^F = c(A_E{}^F)^{-1}(A_E{}^F) = cI$$

SECTION 1.8

42. (a) $(5+3i)/34$ (d) $\text{cis}(-2/3)/4$
(b) $1+i$ (e) $\text{cis}(-7)$
(c) $(3-i)/10$

43. $z\bar{z} = 1$ if and only if $z^{-1} = \bar{z}$

45. (a) $\pm(-1+i)/\sqrt{2}$
(b) $\text{cis}(k\pi/5)$ $k = 1, 3, 5, 7, 9$
(c) $\pm 1$, $\pm i(2^{1/8})\text{cis}(\pi/16)$
(d) $i$, $(\pm 1 + i\sqrt{3})/2$

(e) $(\pm 5)^{1/2} \operatorname{cis}[\frac{1}{2} \arctan(-\frac{3}{4})]$
(f) $250^{1/6} \operatorname{cis}[k\pi/3 + (1/3)\arctan(1/3)]$ $\quad k = 0, 2, 4$

46. (a) $(1, i), (1, -i)$
(b) $(13, 5), (1, 1)$
(c) $(2, 1, 0), ((1 + i/\sqrt{3})/2, 3 - i/\sqrt{3}, 1), ((1 - i/\sqrt{3})/2, 3 + i/\sqrt{3}, 1)$
(d) $(1, -4, 5, 1), (1, 3, 0, 5), ((-3 + \sqrt{21})/4, -2 + \sqrt{21}/2, 0, 1),$
$((-3 - \sqrt{21})/4, -2 - \sqrt{21}/2, 0, 1)$

SECTION 1.9

47. Table of $\langle v_i, v_j \rangle$:

| | $v_2$ | $v_3$ | $v_4$ | $v_5$ | $v_6$ | $v_7$ |
|---|---|---|---|---|---|---|
| $v_1$ | 13 | 24 | 20 | 5 | 2 | 17 |
| $v_2$ | | 41 | 40 | 0 | 4 | 34 |
| $v_3$ | | | 67 | 7 | 5 | 0 |
| $v_4$ | | | | 0 | 5 | −15 |
| $v_5$ | | | | | 0 | 0 |
| $v_6$ | | | | | | 21 |

48. Table of $v_i \times v_j$

| | $v_2$ | $v_3$ | $v_4$ |
|---|---|---|---|
| $v_1$ | (6, −5, −5) | (5, 4, −7) | (9, 2, −10) |
| $v_2$ | | (−5, 13, 7) | (3, 9, 0) |
| $v_3$ | | | (10, −5, −14) |

| | $v_5$ | $v_6$ | $v_7$ |
|---|---|---|---|
| $v_1$ | (−6, 2, 5) | (9, 2, −10) | (11, −72, 25) |
| $v_2$ | (−12, 4, 10) | (−1, −3, 0) | (−41, −123, 0) |
| $v_3$ | (−15, 5, 21) | (0, −7, 0) | (−25, −222, 35) |
| $v_4$ | (−15, 5, 20) | (−2, −6, 0) | (−67, −201, 0) |
| $v_5$ | | (3, −1, 0) | (63, −21, 50) |
| $v_6$ | | | (−5, −15, 0) |

49. $(37, 16, -28)/17$

50. (a) $9x^1 + 2x^2 - 10x^3 = 0$
(b) $12x^1 - 4x^2 - 10x^3 = 0$
(c) $3x^1 - x^2 = 0$
(d) $3x^1 + 9x^2 = 0$

51. $v_1: x = z, 2y = z$
$v_2: x + 3y = 0, z + 4y = 0$
$v_3: y = 0, 5x = 7z$
$v_4: x + 3y = 0, 2z + 5y = 0$
$v_5: z = 0, 3x = y$
$v_6: x = 0, y = 0$
$v_7: x + 3y = 0, 7x + 5z = 0$

52. $7x^1 - x^2 - 2x^3 = 17$

53. $4x^1 + x^2 - 3x^3 = 2$

54. (a) $\langle x, E_2 - E_1\rangle = 0 = \langle x, E_3 - E_1\rangle$
(b) $x^1 + x^2 + 3x^3 = 0, 2x^1 + 2x^2 + 5x^3 = 0$
(c) $x^2 = 0, x^3 = 0$

55. The planes are given by the equations
(a) $x + y + z = 1$ (b) $x = y$ (c) $x = 0$
The intersection of (a) and (b) is given by equations (a) and (b), etc.

56. (a) $x + y = 2z$ (b) $y = z$ (c) $z = 0$

58. The area of a parallelogram of side lengths $a$, $b$ is $ab \sin \theta$, where $\theta$ is (either) included angle.

59. False if $u$ is perpendicular to $v$ and $w$, but $v$ is not perpendicular to $w$.

60. Apply the equation $\langle a, b \times c\rangle = \det\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ to each pair, and observe that there are always two rows the same.

SECTION 1.11

61. (a) open (b) neither (c) closed (d) closed (e) closed
(f) open (g) open (h) closed (i) open (j) open
(k) open

62. $(29, -3, 26)/14$

63. $(111, -22, 111)/34$

64. $(0, 1, 1)/2^{1/2}, (1, -1, 1)/3^{1/2}$
(b) $(0, 1, 0, 1)/2^{1/2}, (1, 0, 1, 0)/2^{1/2}, (-1, -2, 1, 2)/10^{1/2}$
(c) $(0, 1, 0, 0, 0), (0, 0, 0, 1, 0), (0, 0, 2, 0, 1)/5^{1/2}$
(d) $(1, 2, 3, 4)/30^{1/2}, (2, 1, 0, -1)/6^{1/2}, (1, -3, 3, -1)/20^{1/2}$

65. (a) $K$: $(10, -16, 16, 11)/477^{1/2}$
$R$: $(0, 0, 1, 0), (1, 0, 0, 1)/2^{1/2}, (1, 2, 0, -1)/6^{1/2}$
(b) $K$: $(1, 0, -1, 0)/2^{1/2}, (0, 1, 0, -1)/2^{1/2}$
$R$: $(-1, -2, 1, 0)/6^{1/2}, (3, 12, -3, 2)/156^{1/2}$

## Chapter 2

SECTION 2.1

1. (a) does not exist (b) 0 (c) 0 (d) no limit (e) 1 (f) 1 (g) 1 (h) 3

2. No, take $x_n = n^{-1}$

4. Yes

5. 0

SECTION 2.2

8. $-1/2$

9. Form the new sum in this way: at any stage, if the sum is 1, add the first negative term not yet used, and if the sum is less than 1, add positive terms until the sum is 1. The resulting series is

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{4}\underbrace{-\frac{1}{2}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}}\underbrace{-\frac{1}{4}+\frac{1}{16}+\frac{1}{16}+\frac{1}{16}+\frac{1}{16}-\frac{1}{4}}+\cdots$$

The terms come in blocks. The first bracket encloses the first block, the second bracket begins the second block. The $n$th block consists of $2^{n-1}$ copies of

$$-\frac{1}{2^n}+\frac{1}{2^{n+2}}+\frac{1}{2^{n+2}}+\frac{1}{2^{n+2}}+\frac{1}{2^{n+2}}$$

10. Yes. Since the sum of the positive terms is $+\infty$, and the sum of the negative terms is $-\infty$, we can rearrange so that at any stage, if the sum is less than 10,000 we add positive terms until 10,000 is passed, and if the sum is not less than 10,000 add negative terms until 10,000 is passed.

SECTION 2.3

11. (b), (d), (f), (g), (h), (l), (m) converge
(a), (c), (e), (i), (j), (k), (n) diverge

14. (a) $|z| < 1$ (b) $|z| < 1$ (c) all $z$ (d) all $z$ (e) all $z$ (f) $z = 0$ (g) $|z| \leq 1$ (h) $|z| < 1$ (i) $|z| < 1$ (j) $|1 + z| < 1$

SECTION 2.7

15. (a) $\pi$ (b) 2/3 (c) 2/3 (d) 4/3

17. (a) 0 (b) 1/16 (c) $e-(1/2e)-3/2$ (d) 1/4 (e) 5/6
(f) 1/3) $\int_0^{\pi/4} [(1+\sec^2\theta)^{3/2}-1]\,d\theta$

18. (a) 1/6 (b) 1/60 (c) 1/10 (d) $\pi/10$

19. (a) $3\pi/16$ (b) 1/48 (c) 1 (d) 1/24

SECTION 2.8

20.

| | $\partial f/\partial x$ | $\partial f/\partial y$ | $\partial f/\partial z$ |
|---|---|---|---|
| (a) | $yz$ | $xz$ | $xy$ |
| (b) | $y\cos(xy)$ | $x\cos(xy)$ | |
| (c) | $y^z x^{(y^z-1)}$ | $x^{y^z} z y^{z-1} \ln x$ | $x^{y^z} y^z \ln x \ln y$ |
| (d) | $2xy+y^2$ | $x^2+2yx$ | |

21. $x^{x^x}[x^{x-1}+x\ln x+x^x(\ln x)^2]$

23. Since $\nabla h=(\partial h/\partial x^1,\ldots,\partial h/\partial x^n)$ for any function $h$, we need only show that

$$\frac{\partial}{\partial x^i}(fg)=f\frac{\partial g}{\partial x^i}+g\frac{\partial f}{\partial x^i}$$

The proof is just as for functions of one variable.

24. By Exercise 23

$$0=\nabla\left(f\cdot\frac{1}{f}\right)=\frac{1}{f}\nabla f+f\nabla\left(\frac{1}{f}\right)$$

25. 5/9

26. $10^{1/2}/(1+10^{1/2})$

SECTION 2.9

29. (a) 0 (b) 0 (c) 1 (d) 0 (e) 1

30. (b), (c), (f) converge; (a), (d), (e) diverge

SECTION 2.11

31. (a) $x_n=(\frac{1}{2})(x_{n-1}+a/x_{n-1})$
(b) $x_n=2x_{n-1}/3+a/3x_n^{\,2}$

32. (a) $x_n = \frac{2}{3} x_{n-1} - \frac{1}{3} \frac{x_{n-1}^2 + 2x_{n-1} + 3}{3x_{n-1}^2 + 2x_{n-1} + 1}$

(b) $x_n = \frac{x_{n-1}^2 - 1}{2x_{n-1} - 1}, x_1 = ?$

(c) $x_n = x_{n-1} - \frac{x_{n-1}^3 - 2x_{n-1}^2 - 3x_{n-1} + 2}{3x_{n-1}^2 - 4x_{n-1} - 3}$

(d) $x_n = \frac{4}{5} x_{n-1} + 4 \frac{4x_{n-1} + 5}{5x_{n-1}^4 - 1}$

33. (a) all points except on the line $x = 0$
(b) i) all except $(1, -1)$
(ii) all points
(iii) no points

34. $0 = \frac{d}{dx} F(x, g(x)) = \frac{\partial F}{\partial x}(x, g(x)) + \frac{\partial F}{\partial y}(x, g(x))g'(x)$

35. (a) $-(xy + \tan xy)/x^2$
(b) $-\sin(x+y)/(1 + \sin(x+y))$
(c) $-y/x$
(d) $-ye^{xy}/(xe^{xy} - 1)$

## Chapter 3

SECTION 3.1

1. (a) $ce^{ct}$
(b) $(-\sin t, \cos t, 1)$
(c) $(-a \sin t, b \cos t)$
(d) $(2t, 3t^2)$
(e) $(1, 2t, 3t^2)$
(f) $(\cos t, -\sin t, 0)$

2. (a) $|c| \exp[(\text{Re}c)t]$, arg $c$
(b) $2^{1/2}, \pi/2$
(c) $(a^2 \sin^2 t + b^2 \cos^2 t)^{1/2}$
(d) $|t|(4 + 9t^2)^{1/2}$, $\text{arc}\cos(4t + 18t^2)/2\,|t|[(4 + 9t^2)(1 + 9t^2)]^{1/2}$
(e) $(1 + 4t^2 + 9t^4)^{1/2}$
(f) $1 \;\; \pi/2$

3. The tangent to (a) at the point $e^{c\tau}$ is parallel to the tangent to (b) at the point $(a \cos t, b \sin t)$ precisely when

$$\tau = \frac{1}{\operatorname{Im} c} \arctan\left(\frac{b}{a} \tan t\right)$$

4. Never

5. $(ab)^{1/2}$

6. 2

7. $(-1, 1), (-1, 1)$

8. $\min(1/a, 1/b, 1/c)$

| 10. | | Eigenvalues | Eigenvectors |
|---|---|---|---|
| | (a) | 11.516 | (0.685, 0.729) |
| | | −4.516 | (0.729, −0.685) |
| | (b) | 14 | (1, 1) |
| | | 10 | (−1, 1) |
| | (c) | $3 + 4(2)^{1/2}$ | (0.383, 0.924) |
| | | $3 - 4(2)^{1/2}$ | (0.924, −0.383) |
| | (d) | $1 + 2(10)^{1/2}$ | (0.585, 0.811) |
| | | $1 - 2(10)^{1/2}$ | (0.811, −0.585) |
| 11. | (a) | $2 + 2^{1/2}$ | (−0.383, 0, 0.924) |
| | | 1 | (0, 1, 0) |
| | | $2 - 2^{1/2}$ | (0.924, 0, 0.383) |
| | (b) | 7.411 | (−0.501, −0.382, −0.777) |
| | | 0.313 | (0.838, −0.438, −0.325) |
| | | −1.724 | (−0.216, −0.814, 0.540) |

SECTION 3.2

12. $x - x^3/3 + x^5/5,\ 1 - x + x^2 - x^3 + x^4 - x^5$
13. 0.1987
14. 1.7320
16. $[-0.0312, 0.0322],\ |x| \leq 0.1$
17. $|x| \leq 0.125$

SECTION 3.4

18. (a) $y = (-1/2)\exp(-x^2) + 3/2$
    (b) $y = -x \cos x + \sin x$
    (c) $x = t^2/2,\ y = t^3/3 + 1,\ z = t^4/4$
    (d) $z = -ie^{it} + (1 + i)^2 t^3/3 + 1 + i$

19. (a) $y = (c - x)^{-1}$
(b) $\tan y + \sec y = K(\tan x + \sec x)$
(c) $x^3 + y^3 = C$
(d) $y = \sin(x - (1/3)x^3 + C)$
(e) $y = K \int \exp(t^2/2)\,dt + C$
(f) $y = C \exp(x + x^3/3)$
(g) $y = [\ln(1 - x) - x - c]^{-1}$
(h) $y = -\ln(c - e^x)$
(i) $\tan y + \sec y = K \exp(-2 \cos x)$

20. (a) $y = \exp(-x^2/2) \int_0^x \exp(t^2/2)\cos t\,dt$
(b) $y = (\sec x - \cos x)/2$
(c) $y = x - \exp(-x^2/2) \int_0^x \exp(t^2/2)\,dt$
(d) $y = \exp(f(x)) \int_0^x \exp(-f(t) - 2it)\,dt + \exp(1/1 - i)$
where $f(x) = (\exp(1 - i)x)/(1 - i)$
(e) $y = \ln(x + e - 1)$

21. (a) $y = -e^x/2 + e^{-x}/6 + e^{2x}/3$
(b) $y = e^t(\cosh\sqrt{2}t + \sinh\sqrt{2}t/\sqrt{2})$

22. (a) $$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = c_1 \exp(4 + 2i)t \begin{pmatrix} i \\ 1 \end{pmatrix} + c_2 \exp(4 - 2i)t \begin{pmatrix} 1 \\ i \end{pmatrix}$$
(b) $a \neq 0$
$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = c_1 \exp(1 + \sqrt{a})t \begin{pmatrix} 1 \\ -\sqrt{a} \end{pmatrix} + c_2 \exp(1 - \sqrt{a})t \begin{pmatrix} 1 \\ \sqrt{a} \end{pmatrix}$$
$a = 0$
$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} c_2 e^t \\ e^t(-c_2 t + c_1) \end{pmatrix}$$
(c) $a \neq 0$
$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = c_1 e^t \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} + c_2 \exp(1 + \sqrt{2a})t \begin{pmatrix} 1 \\ a/\sqrt{2a} \\ a/\sqrt{2a} \end{pmatrix} + c_3 \exp(1 - \sqrt{2a})t \begin{pmatrix} 1 \\ -a/\sqrt{2a} \\ -a/\sqrt{2a} \end{pmatrix}$$
$a = 0$
$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = [(c_2 + c_3)te^t + c_1 e^t] \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + c_2 e^t \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + c_3 e^t \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

23. (a) $c_1 = \exp[(1 - i)/2]$
(b) $c_1 = 1/2,\ c_2 = -(1 - \sqrt{2a}/a)/4,\ c_3 = (-1 - \sqrt{2a}/a)/4$
(c) $y_1 = [(1 + i)\exp(1 - i)t + (1 - i)\exp(1 + i)t]/2$
$y_2 = [(1 + i)\exp(1 - i)t - (1 - i)\exp(1 + i)t]/2i$

(d)
$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \frac{9\sqrt{17}-17}{34} \exp(1+\sqrt{17})t/2 \begin{pmatrix} \frac{5+\sqrt{17}}{4} \\ \frac{5+\sqrt{17}}{4} \\ 1 \end{pmatrix}$$

$$- \frac{9\sqrt{17}-17}{34} \exp(1-\sqrt{17})t/2 \begin{pmatrix} \frac{5-\sqrt{17}}{4} \\ \frac{5-\sqrt{17}}{4} \\ 1 \end{pmatrix}$$

24. (a)
$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = c_1 e^{-t} \begin{pmatrix} 1 \\ -1 \end{pmatrix} + c_2 e^{5t} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

(d)
$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = c_1 e^{2t} \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix} + c_2 e^{4t} \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} + c_3 e^{-2t} \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$$

(e)
$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = c_1 \exp(6 + i3\sqrt{5})t \begin{pmatrix} 1 \\ \frac{2+i3\sqrt{5}}{7} \end{pmatrix}$$

$$+ c_2 \exp(6 - i3\sqrt{5})t \begin{pmatrix} 1 \\ \frac{2-i3\sqrt{5}}{7} \end{pmatrix}$$

(f) $y_1 = \frac{1}{2}(c \exp(4-7i)t + \bar{c} \exp(4+7i)t)$

$$y_2 = \frac{1}{2i}(c \exp(4-7i)t - \bar{c} \exp(4+7i)t)$$

(g) $y_1 = \mathrm{Re}(c \exp(3-2i))$
$y_2 = \mathrm{Im}(c_1 \exp(3-2i))$
$y_3 = \mathrm{Re}(c_2 \exp(1-2i))$
$y_4 = \mathrm{Im}(c_2 \exp(1-2i))$

(h)
$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = e^t(c_3 t^2/2 + c_2 t + c_1)E_1 + e^t(c_3 t + c_2)E_2 + c_3 e^t E_3$$

(i) $\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = c_1 e^t E_1 + c_2 e^t E_2 + e^t[(c_1 - c_2)t + c_3]E_3$

SECTION 3.7

26. $y = e^{-x}/2 \int t^2 e^t \, dt + c_1 + c_2 e^{-x}$
    $y = x^2/2 - x + c_1 + c_2 e^{-x}$

27. (a) $c_1 e^{2t} + c_2 e^{-2t} - 1/4$
    (b) $c_2 e^x + c_3 e^{-x} - c_1 - (x^3 + 6x)/3$
    (c) $c_1 e^{-x} + c_2 e^{-2x} + (\sin x - 3 \cos x)/10$
    (d) $c_2 e^x + c_1 x$
    (e) $c_1 x^2 + c_2 x^3 + (\ln x)(-x^2 + x^3)$

28. $y = 4x^2/9 + 5/9x + 2x^2 \ln x/3$

29. $y = c_1 x + c_2 x \int \exp[(1 - t)e^t]/t^2 \, dt$

30. $y = e^{-1}\{\exp[(1 - x)e^x] + x \int e^t \exp[(1 - t)e^t] \, dt\} + x - 1$

# Chapter 4

SECTION 4.1

1. $x(\theta) = (\cos \theta, 0, \sin \theta) \qquad 0 \leq \theta \leq 2\pi$

2. $x(\theta) = \dfrac{1 + \sqrt{5}}{2} (\cos \theta, \sin \theta, 1) \qquad 0 \leq \theta \leq 2\pi$

3. $\theta = \arccos(r^{-1} - 1) \qquad r \geq \frac{1}{2}$ parametrizes the curve in the upper half-plane by taking $0 \leq \theta < \pi$; in the lower half-plane by taking $-\pi < \theta \leq 0$.

5. $z(t) = a \cos t e^{it/b}$
   $= e^{it/b}(-b \sin t + i \cos t)/(\cos^2 t + b^2 \sin^2 t)^{1/2}$

6. (a) $(1 - t, t)$
   (b) $(1 + t, \sin 1 - t \cos 1)$
   (c) $(1 + t, 1 - t, t)$
   (d) $(2a, at, t)$
   (e) $(t, 1, t)$
   (f) $(1 + 2t, -2t, 1 + t)$

7. (a) $x$ axis
   (b) $y$ axis
   (c) $y$ axis
   (d) $x = y, z = 0$

SECTION 4.2

8. (a) $$\frac{ds}{d\theta}=\frac{(1+2a\cos\theta+a^2)^{1/2}}{(1+a\cos\theta)^2}$$

(b) $$\frac{ds}{d\theta}=(5+4\cos\theta)^{1/2}$$

(c) (6*a*) $s=(1-e^{-t})/2^{1/2}$

(6*b*) $$\frac{ds}{dx}=\frac{(x^4+\cos^2(1/x))^{1/2}}{x^2}$$

(6*d*) $ds=\int(a^2+\cos^2(\theta/2))^{1/2}\,d\theta$
(6*f*) $s=\int(8t^2+1)^{1/2}\,dt$

(7*a*) $$s=\int_{-1}^{y}\frac{(t^2+1)^{1/2}}{t+1}\,dt$$

10. (a) $a_N=2^{1/2}e^t=a_T$

(b) $$a_T=\frac{4+18t^2}{(4+9t^2)^{1/2}}\operatorname{sgn}t,\quad a_N=3\,|t|\,\frac{(9t^2+16)^{1/2}}{4+9t^2}$$

(d) $a_N=[(1-\sin t)/2]^{1/2}$
$a_T=-[(1+\sin t)/2]^{1/2}$

11. (a) $a_N=|\sin t|(2/2+\cos 2t)^{1/2}$
$a_T=-\sin 2t/(2+\cos 2t)^{1/2}$
(c) $a_T=t/(2+t^2)^{1/2}$, $a_N=[(t^4+5t^2+8)/(t^2+2)]^{1/2}$

SECTION 4.5

12. (a) $y=-xy'$
(b) $xy'+y=0$
(c) $x\exp(-y/xy')=1$
(d) $\sin x(\sin y+xy'\cos y)=x\sin y\cos x$
(e) $y=\exp(x+y)\,y'/y(1+y')$
(f) $1+y'=0$

13. (a) $y=cx$
(b) $x+y^2/2=c$
(c) $2x+y^2=c$
(d) $x=c\exp[-\int^y(t+\sin t)^{-1}]\,dt$
(e) $c(y-1)=\csc(\pi/4-x)$

14. $x^2 + (y - c)^2 = c^2$
$(y^2 - x^2)y' + 2xy = 0$

15. $\dfrac{(x+y)^2}{2a^2} + \dfrac{(y-x-1)^2}{2a^2-1} = 1$

16. $y^2 = cx$

17. (a) $x^2 - y^2 = c$ (b) $y^2 - x^2 = c$ (c) $y = x + c$

18. (a), (b) $x^2 - y^2 + 2\sqrt{3}xy = c$ (c) $y = \dfrac{-x}{\sqrt{3}} - \dfrac{4}{3c} \ln\left(x - \dfrac{1}{c\sqrt{3}}\right)$

19. (a) $xy = 0$ (b) $xy = 0$ (c) $xy = 0$ (d) $\sin x = 0$
(e) $(x + y)y = 0$

20. (a) $y = \pm 1$
(d) $y = \pm e^x$
(e) $r = 1, r = \cos\theta$
(f) $\theta = 0, r = \pm 1$

SECTION 4.6

21. (a) $xy = c$ (b) $x^2 + y^2 = c^2$ (c) $xz = c, yz = d$
(d) $(x, y, z) = (ae^t, b - t, ce^t)$

22. (a) $\left(1 + i\dfrac{\arg z}{\ln|z|}\right)z$ (b) $iz$ (c) $(x, -x^2/2)$ (d) $(1, y, \sqrt{1 - z^2})$

23. (a) $(-x, 1, -z)$
(b) $((1+t)x, (1+t)y, 2t(x^2 + y^2))/(1+t)^2)$
(c) $(0, 1, -z\tan t)$
(d) $(-x, -y, -z(1 + \tan t))$

24. (a) $\exp(t^2/2)(x_0, y_0, z_0)$
(b) $(x_0\cos t - y_0 \sin t, y_0 \cos t + x_0 \sin t, t + z_0)$
(c) $(x_0 e^{-t}, y_0 e^{-t}, z_0 e^t)$

## Chapter 5

SECTION 5.1

1. (a) $|x| < 1/2$ (b) $|x| > 1$ (c) $x < 0$ (d) $-19 < x < -17$
(e) never (f) $|x| < 1$

2. (a) $|z| < 1$ (b) all $z$ (c) $\operatorname{Im} z > 0$

3. (a) both (b) both (c) both (d) integrated for all $x$, differentiate for $x/2\pi$ not an integer

4. (a) $\sum_{n=0}^{\infty} 2(n+1)(-x)^{2n}$

(b) $2\sum_{n=1}^{\infty} nx^{n-1}$

(c) $\sum_{n=0}^{\infty} \frac{x^{2i+1}}{(2i+1)i!}$

(d) $\sum_{n=0}^{\infty} (-1)^n \frac{t^{4n+1}}{4n+1}$

SECTION 5.3

5. (a) $-e^{2x} + xe^{2x} + e^x$
(b) $(3/4)e^{-x} + 2xe^{-x} + (1/4)e^{3x}$
(c) $e^{2x} - 2xe^{2x} + (5/2)x^2e^{2x}$
(d) $3e^x - xe^x$

(e) $\frac{1}{2}\,\mathrm{Re}[(1-2i)e^{ix} + (-1-i)xe^{ix}]$

(f) $(6/5)e^x - (7/40)xe^x - (1/40)xe^{-3x}$
(g) $(1/2)[\exp(2^{1/2}x) + \exp(-2^{1/2}x)] - e^{-x}$

9. $x + \frac{x^4}{12} + \frac{x^7}{504} + \frac{x^{10}}{40360}$

10. $\frac{x^3}{6} + \frac{x^8}{2016}$

11. (a) $a_{n+2} = \frac{2a_{n+1}}{n+2} - \frac{a_n}{(n+1)(n+2)}$

$|a_n| \leq \frac{3K}{n!}$

(b) $a_{n+2} = \frac{2a_{n+1}}{n+2} - \frac{a_{n-1}}{(n+1)(n+2)} + \frac{1}{n!}$

$|a_n| \leq \frac{4K}{[n/2)!}$

(c) $a_{n+k} = \dfrac{-a_k}{(n+1)\cdots(n+k)}$

$|a_n| \leq \dfrac{K}{n!}$

(d) $a_{n+2} = \dfrac{k^2 a_n}{(n+2)(n+1)}$

$|a_n| \leq \dfrac{K}{n!}$

(e) $a_{n+1} = \dfrac{a_{n-1}}{n+1}$

$|a_n| \leq \dfrac{K}{[n/2)!}$

SECTION 5.5

14. (a) $\sum_{n=0}^{\infty} \dfrac{z^{2n}}{n!}$

(b) $\sum_{n=0}^{\infty} \dfrac{(1+i)^n - (1-i)^n}{2in!} z^n$

(c) $\sum_{n=0}^{\infty} \left[ \sum_{k=0}^{[n/2]} \dfrac{(-1)^k}{(2k)!} \right] z^n$

(d) $\sum_{n=0}^{\infty} \dfrac{x^{2n+1}}{(2n+1)n!}$

(e) $\sum_{n=1}^{\infty} \left[ \sum_{j=1}^{n} \sum_{i=0}^{j-1} \dfrac{1}{j(n-j)!\,i!} \right] x^n$

(f) $\sum_{n=0}^{\infty} \dfrac{z^{2n}}{(2n)!}$

(g) $\sum_{n=0}^{\infty} \dfrac{z^{2n+1}}{(2n+1)!}$

16. $\cos(z+w) = \cos z \cos w - \sin z \sin w$
$\cosh(z+w) = \cosh z \cosh w + \sinh z \sinh w$
$\sin(z+w) = \sin z \cos w + \cos z \sin w$
$\sinh(z+v) = \sinh z \cosh w + \cosh z \sinh w$

SECTION 5.7

17. $$a_0\left\{\sum_{n=1}^{\infty}\frac{1}{(2n+1)!}\prod_{j=0}^{n-1}[k(k+1)-(2j)(2j+1)]x^{2n}+1\right\}$$
$$+a_1\left\{\sum_{n=1}^{\infty}\frac{1}{(2n+1)!}\prod_{j=0}^{n-1}[k(k+1)-(2j+1)(2j+2)]x^{2n+1}+x\right\}$$

18. $$a_0\left\{\sum_{n=1}^{\infty}\left[\frac{2^n}{(2n)!}\prod_{j=0}^{n-1}(2j-k)\right]x^{2n}+1\right\}$$
$$+a_1\left\{\sum_{n=1}^{\infty}\left[\frac{2^n}{(2n)!}\prod_{j=0}^{n-1}(2j+1-k)\right]x^{2+1}+x\right\}$$

19. (a) $a_0=1, a_1=2, a_{n+2}=\dfrac{1}{(n+1)(n+2)}\sum_{i+j=n}\dfrac{a_i}{j!}$

(b) $y=x^2/4$ or $y=0$
(c) no solution
(d) $y=(2x^2-c)^{-1}=-(1/c_1)\sum_{n=0}^{\infty}(2/c_1)^n x^{2h}$

20. (a) 10
(b) 460
(c) 3

21. (a) $k$ odd integer, $a_0=0$ or $k$ even integer, $a_1=0$

# Chapter 6

SECTION 6.1

1. (a) $\hat{f}(0)=\pi^2/3$

$$\hat{f}(n)=(-1)^n\frac{2}{n^2}$$

(b) $\hat{f}(5)=\hat{f}(-5)=1/32$
$\hat{f}(3)=\hat{f}(-3)=5/32$
$\hat{f}(1)=\hat{f}(-1)=10/32$
$\hat{f}(n)=0$ all other $n$

(c) $\hat{f}(n)=\dfrac{(-1)^n}{2\pi i}\dfrac{e^{i\mu\pi}-e^{-i\mu\pi}}{\mu-n}=\dfrac{(-1)^n}{\pi}\dfrac{\sin(\mu\pi)}{\mu-n}$, $\mu$ not an integer

(d) $\hat{f}(0) = \pi/8$
$\hat{f}(n) = 0$ if $n = 4k$
$= 1/\pi n^2$ if $n$ is odd
$= 2/\pi n^2$ if $n = 4k + 2$

(e) $\hat{f}(n) = 0$ $n$ odd
$= 2/\pi(1 - n^2)$ $n$ even

(*f*) $\hat{f}(1) = (1 - i)/2$
$\hat{f}(-1) = (1 + i)2$
$\hat{f}(n) = 0$ all other $n$

(g) $\hat{f}(0) = 1/2$
$\hat{f}(n) = 0$ $n$ odd or $n = 4k$, $k \neq 0$
$\hat{f}(n) = -2/\pi in$ $n = 4k + 2$

(h) $\hat{f}(n) = (-1)^n \dfrac{e^\pi - e^{-\pi}}{2\pi(1 - in)}$

(i) $\hat{f}(n) = \dfrac{-2 + 2ine^\pi(-1)^n}{1 + n^2}$

2. (a) $(\text{Re } z)^3 + (\text{Im } z)^3$

(b) $\dfrac{2}{3} \sum_{n=0}^{\infty} \left(\dfrac{-1}{6}\right)^n (r^2e^{2i\theta} + r^2e^{-2i\theta})^n$

(c) $\dfrac{1}{\pi} \sum_{n=-\infty}^{\infty} (-1)^n \dfrac{\sin \mu\pi}{\mu - n} r^{|n|}e^{in\theta}$

(d) $\dfrac{\pi}{8} + \dfrac{1}{\pi} \sum_{n=-\infty}^{\infty} \dfrac{r^{|2n+1|}}{(2n+1)^2} e^{i\theta(2n+1)} + \dfrac{1}{2\pi} \sum_{n=-\infty}^{\infty} \dfrac{r^{2|2n+1|}}{(2n+1)^2} e^{2i\theta(2n+1)}$

(e) $\dfrac{2}{\pi} \text{Re}\left[\left(\dfrac{1}{z} - z\right) \ln \dfrac{1+z}{1-z}\right]$

(f) $(1 + z)^2$

SECTION 6.2

3. (a) $-\dfrac{1}{2} + \dfrac{1}{\pi} \text{Im} \ln\left(\dfrac{1+z}{1-z}\right)$

(b) $\dfrac{-1}{2}(z^2 + \bar{z}^2)$

(c) $\dfrac{2\pi^2}{3} + 2\sum_{n\neq 0}(-1)^n \dfrac{r^{|n|}e^{in\theta}}{n^2}$

(d) $\dfrac{1}{\pi}\sum_{n=-\infty}^{\infty}(-1)^n \dfrac{\sin \mu\pi}{\mu - n} r^{|n|}e^{in\theta}$

(e) $(1+z)^2$

4. (a) $r\sin\theta + r^2\cos 2\theta$

(b) $\dfrac{i}{\pi}\sum_{n\neq 0}\dfrac{r^{|n|}}{|n|}\left(\dfrac{\pi^2}{n^2}(-1)^{n-1} + \dfrac{2(-1)^n - 2}{n^3}\right)e^{in\theta}$

(c) $\dfrac{2}{\pi}\operatorname{Im}\sum_{n=0}^{\infty}\dfrac{z^{2n+1}}{(2n+1)^2} + \dfrac{1}{2}$

SECTION 6.3

5. (a) $(35/2 + 28\cos 2\theta + 14\cos 4\theta + 4\cos 6\theta + (1/2)\cos 8\theta)/64$

(b) $2i\sum_{j=0}^{[k/2]}\binom{k}{j}(-1)^j \sin(k-2j)\theta$      $k$ odd

$2\sum_{j=0}^{k/2}\binom{k}{j}(-1)^j \cos(k-2j)\theta + (-1)^n\binom{k}{k/2}$      $k$ even

(c) $\dfrac{1}{2} + \dfrac{2}{\pi}\sum_{n=0}^{\infty}\dfrac{\sin(2n+1)\theta}{2n+1}$

(d) $\dfrac{1}{2} - \dfrac{2}{\pi}\sum_{n=0}^{\infty}\dfrac{\sin(4k+2)\theta}{2k+1}$

(e) $(\cos 5\theta + 5\cos 3\theta + 10\cos\theta)/16$

6. See Problem 27, Section 6.5

7.

| | cosine series | sine series |
|---|---|---|
| (a) | $1$ | $\dfrac{4}{\pi}\sum_{n=1}^{\infty}\dfrac{\sin(\pi n x)}{n}$ |
| (b) | $(1-\cos 4\pi x)/2$ | $\sin(2\pi x)$ |
| (c) | $\cos(2\pi x)$ | $\dfrac{4}{\pi}\sum_{n=0}^{\infty}\dfrac{(2n+1)\sin(2n+1)\pi x}{(2n+3)(2n+1)}$ |

(d) $\frac{1}{2}+\frac{2}{\pi}\sum_{n=0}^{\infty}\frac{(-1)^n\cos(2n+1)\pi x}{(2n+1)}$ $\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}(\sin(4n+1)\pi x$

$+ 2\sin(4n+2)\pi x + \sin(4n+3)\pi x)$

(e) $(1-\cos 2\pi x)/2$ $\sin \pi x$

(f) $\frac{1}{4}-\frac{8}{\pi^2}\sum_{n=0}^{\infty}\frac{\cos(4n+2)\pi x}{(4n+2)^2}$ $\frac{4}{\pi^2}\sum_{n=0}^{\infty}(-1)^n\frac{\sin(2n+1)\pi x}{(2n+1)^2}$

(g) $(1+2\cos\pi x-\cos 2\pi x)/2$

$$2\sin\pi x+\frac{4}{\pi}\sum_{n=0}^{\infty}\frac{2n+1}{(2n+3)(2n-1)}\sin(2n+1)\pi x$$

8. $f(\theta)=[f(\theta)+f(-\theta)]/2+[f(\theta)-f(-\theta)]/2$

9. $2\sum_{n=0}^{\infty}[A_{2n}\cos(2n)\theta+B_{2n+1}\sin(2n+1)\theta]$

SECTION 6.4

10. (a) $\sum_{n=1}^{\infty}\left(\frac{\sin(\pi n-1)}{\pi n-1}-\frac{\sin(\pi n+1)}{\pi n+1}\right)\cos\pi nt\sin\pi nx$

(b) $\frac{1}{\pi}\sin\pi t\sin\pi x+\frac{64}{\pi}\sum_{n=1}^{\infty}\frac{n^2}{(4n^2-9)(4n^2-1)}\cos 2\pi nt\sin 2\pi nx$

(c) $\frac{-8}{\pi^3}\sum_{n=0}^{\infty}\frac{1}{(2n+1)^3}\cos(2n+1)\pi t\sin(2n+1)\pi x$

(d) $\frac{1}{\pi}\sin\pi t\sin\pi x+\frac{8}{\pi}\sum_{n=1}^{\infty}\frac{n}{4n^2-1}\cos 2\pi nt\sin 2\pi nx$

(e) $\frac{128}{\pi^2}\sum_{n=1}^{\infty}(-1)^{n-1}\frac{n(n^2-2)}{(4n^2-1)(4n^2-9)}\sin\pi nt\sin\pi nx$

11. (a) $\sum_{n=1}^{\infty}\exp\left(\frac{-\pi^2n^2t}{4L^2}\right)\sin\left(\frac{\pi nx}{L}\right)\left(\frac{\sin(\pi n-L)}{\pi n-L}-\frac{\sin(\pi n+L)}{\pi n+L}\right)$

(b) $\frac{8}{\pi}\sum_{n=1}^{\infty}\exp\left(\frac{-\pi^4n^2t)}{L^2}\right)\left(\frac{n}{4n^2-1}\right)\sin\left(\frac{2\pi nx}{L}\right)$

(c) $\displaystyle \frac{-8L^2}{\pi} \sum_{n=0}^{\infty} \exp\left(\frac{-\pi^2 t}{4L^2}(2n+1)^2\right) \frac{1}{(2n+1)^3} \sin \frac{(2n+1)\pi x}{L}$

(d) $\displaystyle \exp\left(\frac{-\pi^2 t}{4L^2}\right) \sin \frac{\pi x}{L} + 3 \exp\left(\frac{-25\pi^2 t}{4L^2}\right) \sin \frac{5\pi x}{L}$

12. (c) $\displaystyle 1 + (e-1)x + \frac{2-4e}{\pi} \sum_{n=0}^{\infty} \exp(-\pi^2(2n+1)^2 t) \frac{\sin(2n+1\pi x)}{2n+1}$

$$+ \sum_{n=1}^{\infty} \exp(-\pi^2 n^2 t) \frac{n(2-e(-1)^n)}{1+n^2+\pi^2} \sin \pi n x$$

14. (a) $\displaystyle \frac{L}{\pi} \exp\left(\frac{-\pi^2 t}{4L^2}\right) \sin \frac{\pi x}{L}$

(b) $\displaystyle -\frac{2L}{\pi} \sum_{n=1}^{\infty} \exp\left(\frac{-\pi^2 n^2 t}{L^2}\right)\left(\frac{1}{n(4n^2-1)}\right) \sin \frac{2\pi n x}{L}$

15. The general solution is of the form

$$\sum_{n=1}^{\infty} (A_n \sin(n^2+1)^{1/2} t + B_n \cos(n^2+1)^{1/2}) \sin nx$$

where the $A_n$, $B_n$ are determined by the sine series of the initial data.

16. On the interval $[-\pi, \pi]$

$$\sum_{n=-\infty}^{\infty} (e^{2inx} - 1)(A_n + B_n e^{int})$$

where the $A_n$, $B_n$ are determined by the Fourier series of the initial data.

SECTION 6.5

17. (a) $27\pi/64$

(b) $\displaystyle \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{n^2}{(n^2-\mu^2)^2} = \frac{2\mu\pi - \sin 2\mu\pi}{2\mu}$

(c) $\displaystyle 2\pi\left(1 + \sum_{n=1}^{\infty} 2r^{2n}\right) = 2\pi \frac{1+r^2}{1-r^2}$

(d) $2\pi r$

(e) $2\pi \sum_{n=1}^{\infty} \frac{1}{n^2}$

(f) $\frac{2\pi^5}{9} + 16\pi \sum_{n=1}^{\infty} \frac{1}{n^4}$

SECTION 6.6

19. (a) span of $\exp(3i\theta)$, $\exp(-5i\theta)$
    (b) span of $\exp(\pm 3i\theta)$, $\exp(\pm i\theta)$
    (c) span of $e^{\pm i\theta}$

20. (a) $(\sin 5\theta + \cos 5\theta)/576 + A \sin\theta + B\cos\theta$

(b) $\frac{2\pi^2}{27} + 2 \sum_{n\neq 0} \frac{(-1)^n e^{in\theta}}{n^2(9 - n^2 + 6in)}$

(c) $\frac{1}{2\pi} \int_{-\pi}^{\pi} \exp(\cos\theta) \sum_{n=-\infty}^{\infty} \frac{\exp[in(\theta - \phi)]}{in^5 + 1} \, d\phi$

SECTION 6.7

21. (a) $13\pi/4$
    (b) $0 \qquad k \leq n$

$$2\pi \binom{k-1}{k-n} \left(-\frac{1}{4}\right)^{k-n} \qquad k \geq n$$

# Chapter 7

SECTION 7.1

1. (a) $[-y \sin x + z \cos(zx)]\, dx + \cos x \, dy + x \cos(2x)\, dz$
   (b) $-[e^{x+y} \sin(e^{x+y}) + e^y \sin(xe^y)]\, dx - [e^{x+y} \sin(x+y) + xe^y \sin(xe^y)]\, dy$
   (c) $e^{\langle x, a\rangle} \langle dx, a\rangle$
   (d) $\langle dx, e^{\langle x, a\rangle}\rangle + \langle x, e^{\langle x, a\rangle}\rangle \langle dx, a\rangle$
   (e) $(2x + z)\, dy + 2y\, dy + x\, dz$
   (f) $e^{x+y}[1 + x - y]\, dx + e^{x+y}[x - y - 1]\, dy$

   (g) $\sum_{j=1}^{n} \left( \prod_{i\neq j} x^i \right) dx^j$

2. (a) 2/1000 (c) 2/1000 (e) 2/1000$e$
   (b) 1/1000$e$ (d) 2/5000 (f) 1/1000$e$

3. $\|p\|^2/1000$ if $\|p\| < 2$, $\|p\|/500$ if $\|p\| \geq 2$.

SECTION 7.2

4. (a) $\begin{pmatrix} e^y & xe^y \\ ye^x & e^x \end{pmatrix}$ $\quad xy \neq 1$

(b) $\begin{pmatrix} 1 & 1 & 1 \\ x^2 + x^3 & x^1 + x^3 & x^1 + x^2 \\ x^2x^3 & x^1x^3 & x^1x^2 \end{pmatrix}$ $\quad x^1, x^2, x^3$ all different

(c) $\begin{pmatrix} 2x & -2y \\ y & x \end{pmatrix}$ $\quad (x, y) \neq 0$

(d) $\begin{pmatrix} 2x & 2y & 2z \\ -y/x^2 & 1/x & 0 \\ -z/x^2 & 0 & 1/x \end{pmatrix}$ $\quad x \neq 0$

(e) $\begin{pmatrix} 1 & 0 & 0 & & \cdots & 0 \\ -x^2/(x^1)^2 & 1/x^1 & 0 & & \cdots & 0 \\ -x^3/(x^1)^2 & 0 & 1/x^1 & 0 & \cdots & 0 \\ -x^n/(x^1)^2 & 0 & & & \cdots & 1/x^1 \end{pmatrix}$ $\quad x \neq 0$

(f) $x^n \dfrac{\partial(h_1, \ldots, h_n)}{\partial(x^1, \ldots, \partial x^n)} + \begin{pmatrix} & h_1 \\ 0 & \vdots \\ & h_n \end{pmatrix}$ $\quad x^n \neq 0$ and the $h_1, \ldots, h_n$ coordinates.

5. $\dfrac{2}{u^1} \sum_{i=2}^{n} u^i \, du^i + 2\left[u^1 - \sum_{i=2}^{n} \dfrac{(u^i)^2}{u_i^3}\right] du^1$

6. (a) $du/u$

(b) $\dfrac{wv\,du}{1 + v^2 + w^2} + \dfrac{1 + w^2 - v^2}{(1 + w^2 + v^2)^2}\, wu\,dv + \dfrac{1 + v^2 - w^2}{(1 + w^2 + v^2)^2}\, vu\,dw$

(c) $\dfrac{1}{2}\dfrac{1 + v + w}{[u(1 + v^2 + w^2)]^{1/2}}\,du + \dfrac{u^{1/2}w(w - v)}{(1 + v^2 + w^2)^{3/2}}\,dv + \dfrac{u^{1/2}v(v - w)}{(1 + v^2 + w^2)^{3/2}}\,dw$

7. $c/2 \quad c =$ fixed edge

SECTION 7.3

9. (a), (b), (d), (g), (i) are closed
(c), (e), (f), (h), (j) are not closed

10. The real part is exact, the imaginary part is not.

11. (a) $e^y$ (c) $-1/y^2$ (e) $\exp(-x-y)$
(b) $e^y/y$ (d) $1/x$ (f) $1/\cos x$

SECTION 7.4

12. (a) 0
(b) 1
(c) $-e\cos 1-(e^2\sin 1)/4-e^2/4+5/4$
(d) $-t-t^2/2\Big|_a^b$
(e) $-a^2/2-2k^2a^5/5$
(f) 21/2

13. (b), (c), (f) are conservative
(a), (d), (e), (g) are not conservative

SECTION 7.5

14. (a) 0 (b) 1

15. (a) 0
(b) $\dfrac{\pi}{2ab}\left[\dfrac{1}{b^2}-\dfrac{1}{a^2}\right]$
(c) 18
(d) $e^6(\cos^2 2)(\sin 2)$
(e) $\pi$

16. (a) $1+\ln 2^{1/2}-2^{-1/2}$
(b) Let $\sin\theta_0=(5^{1/2}-1)/2$. The area is $(3\sin 2\theta_0+\theta_0-\sin^3\theta_0)/6(2)^{1/2}$
(c) 16
(d) If $n$ is even there is no inside. If $n$ is odd, the area is

$$\frac{\pi n!}{2^{2(n-1)}\left[\left(\frac{n-1}{2}\right)!\right]^2}$$

SECTION 7.6

17. (a) $8a^4$
(b) $\infty$
(c) $(2\pi+5\sqrt{3})/3$
(d) $(e^{4\pi}-1)/4$
(e) 4/3

18. (a) 2 (b) $1-2t-3t^2$ (c) $2x+2y$ (d) 0

SECTION 7.7

19. (a) $2^{1/2}\pi$ (b) $3\pi/2^{3/2}$ (c) $e^i[5i-3]/6$ (d) $\pi i\cos(1/2)/2$
(e) $2\pi/3$ (f) $2\pi/(1-\alpha^2)^{1/2}$ (g) $\pi e^{-a}(1+a)/2a^3$
(h) $\pi[\cos(\sqrt{2}/2)+\sin(\sqrt{2}/2)]/\sqrt{2}\exp(-\sqrt{2}/2)$ (i) $\pi/3$ (j) $\pi/e$
(k) $\pi[2\sin(\pi/10)+2\sin(3\pi/10)+1]/5$

## Chapter 8

SECTION 8.1

1. (a) $\infty$ (b) $2\pi$ (c) $\pi/2$ (d) $4\pi/3abc$

2. (a) $2\pi/495$ (b) 0 (c) Let $A = a^{-2}$, $B = b^{-2}$. The integral is

$$\pi(AB)^{1/2}[-5(A^3 + B^3) - 3AB(A + B) + 24A^2 + 6AB + 24B^2]/3(2)^6$$

3. $k\pi a^2 r^6/6$

4. $4\pi(\ln 2 - 1/2)$

5. (a) $(ye^{-t} + tz - t^2x, y(1 - t^2) + (t - 1)(ze^t - txe^t),$
   $-z(1 + t^2) + (1 + t)(x - tye^t))$
   (b) $-3t^2$
   (c) $k(1 - t^3)^{-1}$
   (d) $\infty$

6. (a), (c) are incompressible.

7. (a) $4\pi/3$ (b) $4\pi/3$ (c) $8\pi/3$

SECTION 8.2

9. (a) $(e^t(1 - t) + e^{-t}t(t + 1), -1, te^t(1 - t) - e^{-t})/1 - t^3$
   (b) $(1, -1, 1)$
   (c) $-(1, 1, 1)$
   (d) $(-1, 2z, 0)$
   (e) $(e^{t/2}(1 + t)/2, e^t(1 - t)(e^t(1 - t) + 1), e^{t/2}(2 - t))$
   (f) $(0, 0, y \sin t)$

11. If $M = (a_j^i)$, $\text{div} = a_1{}^1 + a_2{}^2 + a_3{}^3$

$$\text{curl} = (a_3{}^2 - a_2{}^3, a_1{}^3 - a_3{}^1, a_2{}^1 - a_1{}^2)$$

12. 0

SECTION 8.3

13. (b) $ds^2 = (1 + f_x{}^2)\,dx^2 + 2f_x f_y\,dx\,dy + (1 + f_y{}^2)\,dy^2$
    $dS = (1 + f_x{}^2 + f_y{}^2)^{1/2}\,dx\,dy$

14. Tangent plane

(a) $\langle p, (1, -2y, -2z)\rangle = 0$ $(1 + 4y^2)\,dy^2 + 8yz\,dy\,dz + (1 + 4z^2)\,dz^2$

(b) $\langle p, (-x, -y, z)\rangle = 0$ $\dfrac{(2x^2 + y^2)\,dx^2 + 2xy\,dx\,dy + (x^2 + 2y^2)\,dy^2}{x^2 + y^2}$

(c) $\langle p, (-2x, -2y, 1)\rangle = 0$ $(1 + 4x^2)\,dx^2 - 8xy\,dx\,dy + (1 + 4y^2)\,dy^2$

Area element

(a) $(1 + 4y^2 + 4z^2)^{1/2}\,dy\,dz$

(b) $2^{1/2}\,dx\,dy$

(c) $(1 + 4x^2 + 4y^2)^{1/2}\,dx\,dy$

15. (a) $4\pi/3^{1/2}$

16. $\cos\theta = (2v + 2u + uv)/(1 + 4u^2 + v^2)^{1/2}(1 + u^2 + 4v^2)^{1/2}$

18. (a) $2\pi[(1 + a^2)^{3/2} - 1]/3$

(b) $\pi \int_{-\pi}^{\pi} (1 + \sin^2 u)^{1/2}\,du$

(d) $2\pi\left[4 + \frac{1}{\sqrt{1}}\ln\left(\frac{\sqrt{4}+\sqrt{3}}{\sqrt{4}-\sqrt{3}}\right)\right]$

SECTION 8.4

19. (a) $2^{1/2}\pi/4$ (b) 0

20. (a) $e - 2$ (b) $-4\pi/3$ (c) 0

SECTION 8.5

25. (a) $9\pi/2(ab)^{1/2}$ (b) $\pi/3(ab)^{1/2}$ (c) 1/18

28. (a) 0 (b) $-4\pi/3$